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# A full-potential linear-muffin-tin-orbital molecular-dynamics study of $\mathbf{B}_{7}, \mathbf{B}_{10}$ and $B_{13}$ clusters 

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#### Abstract

The structures of $\mathrm{B}_{7}, \mathrm{~B}_{10}$ and $\mathrm{B}_{13}$ boron clusters are studied using the fullpotential linear-muffin-tin-orbital molecular-dynamics method. Seven stable structures for $\mathrm{B}_{7}$ and fifteen for $\mathrm{B}_{10}$ have been obtained. $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{10}$ is the most stable among the 15 structures, but $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{10}$ is not stable. For $\mathrm{B}_{13}$, three degenerate ground-state structures have been found. The potential surface near $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}$ (ground state) and $\mathrm{D}_{6 \mathrm{~h}}-\mathrm{B}_{7}$ is very flat. As a fundamental unit in constructing bigger clusters, $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}$ will change its form easily. The most stable structures for $\mathrm{B}_{7}, \mathrm{~B}_{10}$ and $\mathrm{B}_{13}$ clusters are two-dimensional (quasi-) planar clusters, rather than the three-dimensional ones. General speaking, these clusters obey the 'Aufbau principle'.


## 1. Introduction

Atomic clusters represent a new phase of matter between molecules and solids, and show increasing potential for technological applications; as a result of this, their structures and energetics have been the focus of many experimental and theoretical studies in the past few years [1,2]. The trivalent semiconductor elemental boron has a low density but a very high melting point of $2300{ }^{\circ} \mathrm{C}$ (about $1000{ }^{\circ} \mathrm{C}$ higher than that of silicon) and a high hardness similar to that of diamond. Considerable progress has been made in our understanding of the evolution of equilibrium geometries, the nature of the bonding and the electronic structure for boron. However, unlike boron crystal and boron compounds, the number of experimental and theoretical studies of boron clusters has been rather limited up to now. Hanley et al have measured their appearance potentials and fragmentation pattern by studying the collisioninduced dissociation of ions of boron clusters containing up to 13 atoms [3]. La Placa et al reported the discovery of boron clusters $\left(\mathrm{B}_{n}, n=2-50\right)$ formed by ablation of hexagonal boron nitride with a 532 nm laser [4]. On the theoretical side, small boron clusters were investigated by several authors using different methods [5-12]. Niu et al calculated the equilibrium geometries, binding energies and electronic structures of neutral and charged boron clusters containing up to six atoms [6]. Accurate calculations based on ab initio quantum-chemical methods were carried out for small boron clusters $\mathrm{B}_{n}(n=2-14)$ to determine their electronic and geometric structures [7]. It was found that most of the final structures of the boron clusters $(n \geqslant 10)$ are
composed of two fundamental units: either of hexagonal or of pentagonal pyramids; this is the so-called 'Aufbau principle'. The 'Aufbau principle' leads to four topological groups of stable boron clusters, i.e., convex, quasi-planar, tubular and spherical clusters. Applying the 'Aufbau principle', Boustani and Quandt constructed boron nanotubules of $B_{24}$ and $B_{32}$, with different numbers of rings, and $\mathrm{B}_{48}$ using ab initio quantum-chemical and density functional methods to determine the relation of energy to the number of rings [10, 11]. Using ab initio quantum-chemical methods, Sabra and Boustani studied ground-state energies and geometrical configurations of boron and boron-hydrogen chains; they found that the ground-state energies of the boron chains were comparable to those of the boron clusters [9].

From the references cited above, it is clear that the 'Aufbau principle' is very useful as regards constructing the bigger boron clusters. For example, the decamer $\mathrm{B}_{10}$ results from adding three atoms to the $\mathrm{B}_{7}$ unit. In this case the $\mathrm{B}_{10}$ cluster contains two similar subunits. The convex structure of the first isomer $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{10}$ is characterized by having two top atoms, while the quasi-planar structure of the second isomer $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{10}$ is characterized by having top and bottom atoms [7]. On adding three more B atoms, the $\mathrm{B}_{13}$ cluster will have three similar subunits of $\mathrm{B}_{7}$. In order to obtain deeper insight into the structures of boron clusters and their formation from elemental units, it is worthwhile to carry out more calculations to investigate all of the possible structures for $B_{7}, B_{10}$ and $B_{13}$ clusters.

In recent years, we have studied the structures and energies of small $\mathrm{Si}_{n}, \mathrm{Ge}_{n}$ and $\mathrm{Ga}_{n} \mathrm{As}_{n}$ clusters (up to $n=60$ ) and the interaction between small Si clusters and $\mathrm{NH}_{3}, \mathrm{CO}$ and $\mathrm{H}_{2} \mathrm{O}$ using the full-potential linear-muffin-tin-orbital molecular-dynamics (FP-LMTO MD) method [13-19]. We found a series of new stable and ground-state structures. For example, our calculation found that the stable structure for the $\mathrm{Si}_{60}$ cluster is a distorted truncated icosahedron, with $\mathrm{T}_{\mathrm{h}}$ symmetry. The lower symmetry and four distinct $\mathrm{Si}-\mathrm{Si}$ bond lengths distinguish this structure from that with $\mathrm{I}_{\mathrm{h}}$ symmetry and two $\mathrm{C}-\mathrm{C}$ bond lengths in $\mathrm{C}_{60}$ [13]. Our results for $\mathrm{Si}_{n}$ and $\mathrm{Ge}_{n}$ are in good agreement with those from related experiments [20,21]. All of our results show that the FP-LMTO MD method is an accurate way to calculate the structures of small clusters.

In the current paper, eight stable structures of $B_{7}$, fifteen stable structures of $B_{10}$ and some of the $B_{13}$ clusters are presented. Some consideration is given to the construction of bigger boron clusters.

## 2. Method

The FP-LMTO method [22-25] is a self-consistent implementation of the Kohn-Sham equations in the local-density approximation [26]. In this method, space is divided into two parts: nonoverlapping muffin-tin (MT) spheres centred at the nuclei, and the remaining interstitial region. LMTOs are augmented Hankel functions, and are augmented inside the MT spheres but not in the interstitial region [27]. In the LMTO method, one thing that we must do is to calculate the interstitial-potential matrix elements:

$$
\begin{equation*}
V_{i j}^{I}=\int_{I} \phi_{i}(x) V_{I}(x) \phi_{j}(x) \mathrm{d} x \tag{1}
\end{equation*}
$$

where $I$ is the interstitial region, $V_{I}$ is the interstitial potential, $i$ and $j$ are abbreviations for $\nu L$ and $\nu^{\prime} L^{\prime}$, respectively, and $\phi_{i}$ is a LMTO envelope function centred at site $\nu$ with angular momentum $L . L$ is an abbreviation for the angular-momentum quantum numbers $(l, m)$. In different methods, we handle $V_{i j}^{I}$ in different ways. In the FP-LMTO method for clusters, Methfessel and co-workers $[24,25]$ used nonoverlapping MT spheres. They retained nonspherical potential terms inside the MT spheres, but expanded the interstitial potential
$V_{I}(x)$ in a different set of atom-centred Hankel functions. In order to obtain the interstitialpotential matrix elements, we need an accurate representation, valid in the interstitial region, of the product of two Hankel functions centred at the same site or at different sites. That is, we require an expansion of the form

$$
\begin{equation*}
\phi_{i}^{*} \phi_{j}=\sum_{k} C_{k}^{i j} \chi_{k}(x) \tag{2}
\end{equation*}
$$

where $\phi_{i}$ is a LMTO envelope function centred at site $v$ with angular momentum $L . k$ is an abbreviation for ( $v L \alpha$ ), and the index $\alpha$ runs over different locations; $\chi_{k}$ are functions of the charge-density Hankel function set; $C_{k}^{i j}$ are expansion coefficients. The interstitial-potential matrix element $V_{i j}^{I}$ then reduces to a linear combination of integrals of the functions $\chi_{k}$ times the interstitial potential. Because the interstitial potential itself is also expanded in functions of $\chi_{k}$ type, the desired interstitial integral has now been expressed as a linear combination of integrals of products of pairs of Hankel functions; i.e., the three-centre integral has been reduced to a sum of two-centre integrals. Because the products are smooth functions, the coefficients in equation (2) above can be adjusted until the best fit of the values and slopes of the right-hand side to the values and slopes of the products is obtained for all spheres simultaneously by tabulation for the surfaces of the spheres. In the cluster method, the expansion is first calculated for two atoms arranged along the $z$-axis, and the coefficients are tabulated as functions of the interatomic distance. For a general geometry, the expansion is obtained by rotating the tabulated fit using the rotation matrices of the spherical harmonics. The tabulated fit is made by a direct numerical integration and can be made as accurate as desired [23]. The force expression for the FP-LMTO method can be obtained using the Harris energy functional [22,23]. During the optimization of one structure, no restriction is imposed. Starting with one initial geometric configuration, we set up one time step. In each time step, the eigenvalue problem is solved exactly, and the output density is admixed with the input density in the usual way. The nuclei are then moved according to the forces using the Verlet algorithm. We then decompose the mixed density, move each partial density along with its atom, and re-overlap at the new geometry. After many iterations, the maximum of the forces is less than 0.001 au , and the total energy stays nicely constant because the system stays close to self-consistency (so the nearly zero forces agree with the energy minima). The process is stopped when the self-consistency condition is met, and a stable (or sub-stable) final calculated structure is obtained.

The details of how the molecular-dynamics simulation can be performed are described in $[22,23]$. The initial structures are constructed from some consideration of symmetry. In order to have a sufficiently high probability of finding all of the local minima on the potential surface and the ground state, the number of initial structures is as large as possible based on the reasonable guesses of configurations.

## 3. Results and discussion

First of all, we calculated the structures for the dimer $B_{2}$ and trimer $B_{3}$. The calculated bond length of $\mathrm{B}_{2}$ is $1.605 \AA$, in good agreement with the experimental value of $1.59 \AA$. For $\mathrm{B}_{3}$, the final structure is triangular in shape with symmetry $\mathrm{D}_{3 \mathrm{~h}}$, and the resulting bond length is $1.531 \AA$, a little bit shorter than the result in [7]. Considering that their bond length for $B_{2}$ is $1.67 \AA, 0.08 \AA$ larger than the experimental value, we think that our calculated value for $B_{3}$ is reasonable. If the initial structure is an isosceles triangle with the angle of $28^{\circ}$ and equal sides of $1.60 \AA$, our result shows that this initial structure will gradually move to $D_{3 h}$, as found above. The average binding energies for $\mathrm{B}_{2}$ and $\mathrm{B}_{3}$ are 1.407 eV and 3.590 eV respectively.

## 3.1. $B_{7}$

Seven final calculated structures are shown in figure 1, and the bond lengths, binding energies and point groups are given in table 1. These seven final structures are obtained from twenty nine initial structures. One final structure can be obtained from several different initial states. $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}$ (a in figure 1) is the most stable structure (ground state) among these seven final structures; it consists of a quasi-planar hexagon capped by a single atom 4 at a height of $0.64 \AA$ above the plane of atoms $2,3,5,7$, with the atoms 1 and 6 a little bit higher ( $0.18 \AA$ ) than them. Our result of $\mathrm{C}_{2 v}-\mathrm{B}_{7}$ is very similar to that in [7]. The average axial bond length between the apex Si atom and equatorial atoms is $1.71 \AA$, while that between the equatorial atoms is $1.60 \AA$ in [7], and they are $1.69 \AA$ and $1.59 \AA$ respectively for our results. As for $B_{2}$ and $B_{3}$, our results are little bit shorter than those of [7]. There is in fact a final structure with higher


Figure 1. Geometries of the seven final stable structures of the $B_{7}$ cluster.

Table 1. Point groups, bond lengths $(\AA)$ and binding energies $E_{b}$ and $E_{b} / n(\mathrm{eV})$ for $\mathrm{B}_{7}$ clusters.

| Structure | Bond name | Bond length | Binding energy |
| :--- | :--- | :--- | :--- |
| $\mathrm{a}\left(\mathrm{C}_{2 \mathrm{v}}\right)$ | $d_{41}, d_{46}$ | 1.707 | $36.705(5.244)$ |
|  | $d_{42}, d_{43}, d_{45}, d_{47}$ | 1.677 |  |
|  | $d_{57}, d_{23}$ | 1.629 |  |
|  | $d_{12}, d_{15}, d_{63}, d_{67}$ | 1.566 |  |
| $\mathrm{~b}\left(\mathrm{C}_{2}\right)$ | $d_{54}, d_{21}$ | 1.809 | $35.412(5.059)$ |
|  | $d_{64}, d_{72}$, | 1.663 |  |
|  | $d_{36}, d_{37}$ | 1.591 |  |
|  | $d_{56}, d_{17}$ | 1.573 |  |
|  | $d_{41}, d_{52}$ | 1.566 |  |
| $\mathrm{c}\left(\mathrm{quasi}-\mathrm{D}_{5 \mathrm{~h}}\right)$ | $d_{\text {min }}: d_{45}$ | 1.624 | $35.277(5.040)$ |
|  | $d_{\text {max }}: d_{52}$ | 1.804 |  |
| $\mathrm{~d}\left(\mathrm{C}_{\mathrm{s}}\right)$ | $d_{\text {min }}: d_{31}, d_{32}, d_{36}$ | 1.652 | $34.786(4.970)$ |
| $\mathrm{e}\left(\mathrm{C}_{6 \mathrm{v}}\right.$, quasi-D |  |  |  |
|  | $d_{\text {max }}: d_{17}, d_{64}$ | $d_{12}, d_{25}, d_{57}, d_{76}, d_{63}, d_{31}$ | 1.683 |
| $\mathrm{f}\left(\mathrm{C}_{2 \mathrm{v}}\right)$ | $d_{41}, d_{42}, d_{45}, d_{47}, d_{46}, d_{43}$ | 1.624 | $36.375(5.194)$ |
| $\mathrm{g}\left(\mathrm{D}_{2 \mathrm{~h}}\right)$ | $d_{\text {min }}: d_{47}, d_{13}$ | 1.487 | $35.671(5.096)$ |
|  | $d_{\text {max }}: d_{25}, d_{26}$ | 1.775 |  |
|  | $d_{\text {min }}: d_{64}$ | 1.517 | $33.710(4.816)$ |
|  | $d_{\text {max }}: d_{74}$ | 1.619 |  |

symmetry, i.e., $\mathrm{C}_{6 \mathrm{v}}-\mathrm{B}_{7}$ (e in figure 1), with atom 4 only $0.10 \AA$ higher than the hexagon plane. It is almost a $D_{6 h}$ structure (quasi- $\mathrm{D}_{6 \mathrm{~h}}$ ). The total binding energy of $\mathrm{C}_{6 \mathrm{v}}-\mathrm{B}_{7}$ is only 0.33 eV less than that of $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}$. We calculated the total energy of several structures between $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}$, $\mathrm{C}_{6 \mathrm{v}}-\mathrm{B}_{7}$ and $\mathrm{D}_{6 \mathrm{~h}}-\mathrm{B}_{7}$. We found that all of these total energies are very close to that of $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}$. The maximum energy difference is only 0.34 eV in our calculations. So the potential surface is very flat in this area.

There are three three-dimensional structures in our results (b, c and din figure 1). Their total binding energies are $1.29 \mathrm{eV}, 1.43 \mathrm{eV}$ and 1.92 eV less than that of $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}$. In these three three-dimensional structures, $\mathrm{C}_{2}-\mathrm{B}_{7}$ (b in figure 1) is more stable than quasi- $\mathrm{D}_{5 \mathrm{~h}}-\mathrm{B}_{7}$ (c in figure 1) and $\mathrm{C}_{\mathrm{i}}-\mathrm{B}_{7}$ (d in figure 1), while $\mathrm{D}_{5 \mathrm{~h}}-\mathrm{B}_{7}$ is reported to be the optimized 3D structure in [7]. Because these three 3D structures are obtained from 15 different 3D initial structures, it is reasonable to assume that there are no other 3D structures for the $\mathrm{B}_{7}$ cluster.

In figure 1, f and g are two chain planar structures. Their total binding energies are 1.03 eV and 2.99 eV less than that of the ground state $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}$. Although the binding energy of structure g is the lowest, we found that because its structure is very different from the other structures, it is a rather stable isomer. The big change in initial structure around it will not affect the final structure g .

As pointed in [12], the hexagonal pyramid is one of the basic units in the 'Aufbau principle'. According to our results mentioned above, the potential surface near $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}$ and quasi- $\mathrm{D}_{6 \mathrm{~h}}-\mathrm{B}_{7}$ is very flat. When the bigger clusters are constructed from $\mathrm{B}_{7}$ hexagonal pyramids, the form of the pyramids will change easily as a result of the interaction between them.

## 3.2. $B_{10}$

The 15 calculated final structures are shown in figure 2, and their binding energies, point groups and bond lengths for some representative structures are given in table 2. These structures are


Figure 2. Geometries of the fifteen final stable structures of the $\mathrm{B}_{10}$ cluster.
obtained from 45 initial structures. As for $B_{7}$, one final structure can be obtained from several different initial structures. In our results, the ground-state structure is $\mathrm{C}_{2 h}-\mathrm{B}_{10}$ (a) (a in figure 2), a quasi-planar structure which is the same as that found in [7]. Atom 7 and atom 3 are about $0.26 \AA$ above and below the plane of the other atoms. The average of the bond lengths between neighbouring peripheral atoms is $1.60 \AA(1.60 \AA)$; the distance between the top and bottom is $1.61 \AA(1.63 \AA)$; the averages of the bond lengths between the apices and the peripheral atoms along the short and long diagonals of the cluster are $1.75 \AA(1.80 \AA)$ and $1.64 \AA(1.67 \AA)$ respectively, where the values in parentheses are Boustani's results. In [7], it is reported that


Figure 2. (Continued)
the convex structure of $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{10}$ has almost the same binding energy as $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{10}$. However, if we take $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{10}$ as the initial structure, our molecular-dynamics calculation shows that it will automatically change to the quasi-planar structure $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{2}$. Changing the heights of atom 3 and atom 7 in the initial structures, we get the same final structure $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{10}$. So we think that the convex structure of $\mathrm{C}_{2 v}-\mathrm{B}_{10}$ is not stable. At the same time, as two $\mathrm{B}_{7}$ pyramids construct quasi-planar $B_{10}$, the height of the apex atom will be decreased from $0.64 \AA$ (in the $B_{7}$ pyramid)

Table 2. Point groups, bond lengths $(\AA)$ and binding energies $E_{b}$ and $E_{b} / n(\mathrm{eV})$ for $\mathrm{B}_{10}$ clusters. (For some clusters with comparatively small binding energies, just the minimum and maximum bond lengths are shown.)

| Structure | Bond name | Bond length | Binding energy |
| :---: | :---: | :---: | :---: |
| $\mathrm{a}\left(\mathrm{C}_{2 \mathrm{~h}}\right)$ | $d_{98}, d_{910}, d_{42}, d_{46}$ | 1.584 | 57.820 (5.782) |
|  | $d_{37}$ | 1.607 |  |
|  | $d_{105}, d_{65}, d_{81}, d_{21}$ | 1.613 |  |
|  | $d_{93}, d_{47}$ | 1.625 |  |
|  | $d_{72}, d_{76}, d_{38}, d_{310}$ | 1.647 |  |
|  | $d_{57}, d_{17}, d_{53}, d_{13}$ | 1.745 |  |
| $\mathrm{b}\left(\mathrm{C}_{\mathrm{s}}\right)$ | $d_{12}$ | 1.543 | 57.120 (5.712) |
|  | $d_{56}$ | 1.556 |  |
|  | $d_{29}, d_{18}$ | 1.561 |  |
|  | $d_{83}, d_{94}$ | 1.564 |  |
|  | $d_{105}, d_{106}$ | 1.613 |  |
|  | $d_{35}, d_{46}$ | 1.618 |  |
|  | $d_{710}$ | 1.647 |  |
|  | $d_{71}, d_{72}$ | 1.693 |  |
|  | $d_{103}, d_{104}$ | 1.707 |  |
|  | $d_{78}, d_{79}$ | 1.793 |  |
| $\mathrm{c}\left(\mathrm{C}_{\mathrm{s}}\right)$ | $d_{56}$ | 1.538 | 56.717 (5.672) |
|  | $d_{38}, d_{49}$ | 1.557 |  |
|  | $d_{12}$ | 1.560 |  |
|  | $d_{810}, d_{910}$ | 1.571 |  |
|  | $d_{31}, d_{42}$ | 1.590 |  |
|  | $d_{53}, d_{64}$ | 1.682 |  |
|  | $d_{710}$ | 1.720 |  |
|  | $d_{71}, d_{72}$ | 1.735 |  |
|  | $d_{57}, d_{67}$ | 1.801 |  |
|  | $d_{78}, d_{79}$ | 1.836 |  |

to $0.26 \AA$ in $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{10} . \mathrm{B}_{10}$ (b) (b in figure 2) is a new structure never reported before; its binding energy is only little bit smaller than that of the ground state $\mathrm{B}_{10}(\mathrm{a})$. There is only one apex atom (atom 10 in figure 2), and the other atoms are almost on the same plane. The height of the apex atom is about $0.52 \AA$.

Comparing with $\mathrm{B}_{7}$, the number of three-dimensional structures $\left(\mathrm{B}_{10}(\mathrm{c})\right.$ to $\mathrm{B}_{10}(\mathrm{n})$ in figure 2) is greatly increased. Our 12 final 3D structures are obtained from 39 different initial structures. We think that it would be hard to obtain more new 3D structures, especially clusters with comparatively large binding energies. The binding energy of $\mathrm{B}_{10}(\mathrm{c})$ is the largest among all of these 3 D structures, but it is still 1.10 eV smaller than that of the ground state $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{10}$. The binding energy of $B_{10}(n)$ is the smallest among the 3 D structures, 4.37 eV smaller than that of $B_{10}(a)$. In the $B_{7}$ cluster, there is a planar structure of quasi- $\mathrm{C}_{6 \mathrm{~h}}$ symmetry $\left(\mathrm{B}_{7}(\mathrm{e})\right.$ in figure 1), but in $B_{10}$ there is no similar planar structure. If we take this structure as the initial state (height $=0$ for apex atom 3 and atom 7 as mentioned above), it will automatically change to $\mathrm{B}_{10}(\mathrm{a})$.

There is a chain structure in $B_{10}$ clusters $\left(B_{10}(0)\right.$ in figure 2). Its binding energy per atom is $5.0185 \mathrm{eV}, 0.764 \mathrm{eV}$ less than that of the ground state $\mathrm{B}_{10}(\mathrm{a})$. Although the binding energy of $\mathrm{B}_{10}(\mathrm{o})$ is the smallest among the $\mathrm{B}_{10}$ clusters, like $\mathrm{B}_{7}(\mathrm{~g})$ it is a rather stable structure.

Table 2. (Continued.)

| Structure | Bond name | Bond length | Binding energy |
| :---: | :---: | :---: | :---: |
| $\mathrm{d}\left(\mathrm{C}_{2 \mathrm{v}}\right.$, quasi- $\left.\mathrm{D}_{2 \mathrm{~d}}\right)$ | $\begin{aligned} & d_{67}, d_{610}, d_{58}, d_{59} \text { and } \\ & d_{47}, d_{49}, d_{210}, d_{28} \\ & d_{19}, d_{110}, d_{37}, d_{38} \\ & d_{31} \end{aligned}$ | $\begin{aligned} & 1.542 \\ & 1.703 \\ & 1.843 \end{aligned}$ | 55.931 (5.593) |
| $\mathrm{e}\left(\mathrm{C}_{1}\right)$ | $\begin{aligned} & d_{\min }: d_{12}, d_{18}, d_{64}, d_{65} \\ & d_{\max }: d_{710} \end{aligned}$ | $\begin{aligned} & 1.537 \\ & 1.864 \end{aligned}$ | 55.886 (5.589) |
| $\mathrm{f}\left(\mathrm{C}_{\mathrm{s}}\right)$ | $\begin{aligned} & d_{\text {min }}: d_{12} \\ & d_{\text {max }}: d_{38}, d_{48} \end{aligned}$ | $\begin{aligned} & 1.526 \\ & 1.789 \end{aligned}$ | 55.589 (5.559) |
| $\mathrm{g}\left(\mathrm{C}_{2}\right)$ | $\begin{aligned} & d_{\text {min }}: d_{29}, d_{15} \\ & d_{\text {max }}: d_{39}, d_{35} \end{aligned}$ | $\begin{aligned} & 1.556 \\ & 1.743 \end{aligned}$ | 54.773 (5.477) |
| $\mathrm{h}\left(\mathrm{D}_{5 \mathrm{~h}}\right)$ | $\begin{gathered} d_{12} \\ d_{16} \end{gathered}$ | $\begin{aligned} & 1.587 \\ & 1.588 \end{aligned}$ | 54.619 (5.462) |
| $\mathrm{i}\left(\mathrm{C}_{\mathrm{s}}\right)$ | $\begin{aligned} & d_{\min }: d_{92}, d_{103} \\ & d_{\max }: d_{45} \end{aligned}$ | $\begin{aligned} & 1.574 \\ & 1.792 \end{aligned}$ | 54.614 (5.461) |
| $\mathrm{j}\left(\mathrm{C}_{2 \mathrm{v}}\right)$ | $\begin{aligned} & d_{\text {min }}: d_{71}, d_{73} \\ & d_{\text {max }}: d_{13} \end{aligned}$ | $\begin{aligned} & 1.542 \\ & 1.819 \end{aligned}$ | 54.462 (5.446) |
| $\mathrm{k}\left(\mathrm{C}_{2 \mathrm{v}}\right)$ | $\begin{aligned} & d_{\min }: d_{89}, d_{16}, d_{37}, d_{42} \\ & d_{\max }: d_{94}, d_{93}, d_{64}, d_{63} \\ & \text { and } d_{28}, d_{21}, d_{78}, d_{71} \end{aligned}$ | $\begin{aligned} & 1.545 \\ & 1.890 \end{aligned}$ | 54.458 (5.446) |
| $1\left(\mathrm{C}_{\mathrm{i}}\right)$ | $\begin{aligned} & d_{\min }: d_{810}, d_{67} \\ & d_{\max }: d_{19}, d_{13}, d_{24}, d_{25} \end{aligned}$ | $\begin{aligned} & 1.535 \\ & 1.869 \end{aligned}$ | 54.431 (5.443) |
| $\mathrm{m}\left(\mathrm{D}_{5 \mathrm{~d}}\right)$ | $\begin{aligned} & d_{18}, d_{110} \\ & d_{109}, d_{12} \end{aligned}$ | $\begin{aligned} & 1.586 \\ & 1.588 \end{aligned}$ | 53.841 (5.385) |
| $\mathrm{n}\left(\mathrm{C}_{\mathrm{i}}\right)$ | $\begin{aligned} & d_{\min }: d_{53}, d_{13}, d_{46}, d_{42} \\ & d_{\max }: d_{56}, d_{12} \end{aligned}$ | $\begin{aligned} & 1.568 \\ & 1.723 \end{aligned}$ | 53.447 (5.345) |
| $o\left(\mathrm{D}_{2 \mathrm{~h}}\right)$ | $\begin{aligned} & d_{\min }: d_{910}, d_{810}, d_{12}, d_{13} \\ & d_{\max }: d_{87}, d_{97}, d_{42}, d_{43} \end{aligned}$ | $\begin{aligned} & 1.511 \\ & 1.640 \end{aligned}$ | 50.185 (5.019) |

## 3.3. $B_{13}$

In $\mathrm{B}_{7}$ and $\mathrm{B}_{10}$ clusters, the binding energy of the two-dimensional structures is higher than that of the 3D structures. The two-dimensional structures are more stable. For $\mathrm{B}_{13}$, we only studied the two-dimensional structures at this stage.

Five calculated final structures are shown in figure 3, and their binding energies, point groups and bond lengths are given in table 3 .

In these five final structures, $\mathrm{B}_{13}(\mathrm{a})$, (b) and (d) are almost degenerate. The total binding energy of (d) is 0.05 eV greater than that of (a) and 0.09 eV greater than that of (b). The atoms in (a) and (d) are almost on the same plane: in (a), atoms 6 (and 8 ) and 7 are only $0.12 \AA$ and $0.09 \AA$ above the other atoms. In (d), the biggest height difference is less than $0.1 \AA$. The
a


Figure 3. Geometries of the five final stable two-dimensional (quasi-) planar clusters of $\mathrm{B}_{13}$.
structure of $B_{13}(b)$ is very similar to that of (a), except that its atoms are not on the plane but on the surface of a arch, producing a convex structure. For $\mathrm{B}_{13}(\mathrm{e})$, compared with (a) and (d), three more atoms are added on the corner of $\mathrm{B}_{10}(\mathrm{a})$, whose total binding energy is 0.91 eV less than that of $B_{13}(a)$, and the cluster has lost all symmetry. In $B_{13}(e)$, atoms 1 and 3 have moved down by about $0.30 \AA$ and $0.37 \AA$, while atoms 5,11 and 12 have moved up by about $0.39,0.43$ and $0.16 \AA$ respectively; the other atoms are almost on the same plane. General speaking, this cluster is still constructed of three hexagonal pyramids, although they are deformed. However, the ground state $B_{13}(d)$ is a new structure, only containing two hexagonal pyramids (in fact, two quasi- $\mathrm{D}_{6 \mathrm{~h}}$ structures).

Our final structure, $\mathrm{B}_{13}(\mathrm{~b})$, is similar to the convex structure $\mathrm{C}_{2 v}-\mathrm{B}_{13}$ of Boustani, but our structure is flatter. The average of the bond lengths between the peripheral atoms is $1.60 \AA$

Table 3. Point groups, bond lengths $(\AA)$ and binding energies $E_{b}$ and $E_{b} / n(\mathrm{eV})$ for $\mathrm{B}_{13}$ clusters.

| Structure | Bond name | Bond length | Binding energy |
| :---: | :---: | :---: | :---: |
| $\mathrm{a}\left(\mathrm{C}_{2 \mathrm{v}}\right)$ | $d_{67}, d_{78}$ | 1.554 | 77.171 (5.936) |
|  | $d_{51}, d_{510}, d_{94}, d_{913}$ | 1.593 |  |
|  | $d_{12}, d_{1011}, d_{34}, d_{1213}$ | 1.609 |  |
|  | $d_{56}, d_{89}$ | 1.621 |  |
|  | $d_{23}, d_{1112}$ | 1.631 |  |
|  | $d_{16}, d_{106}, d_{48}, d_{138}$ | 1.656 |  |
|  | $d_{26}, d_{116}, d_{38}, d_{128}$ | 1.732 |  |
|  | $d_{27}, d_{37}, d_{117}, d_{127}$ | 1.765 |  |
| $\mathrm{b}\left(\mathrm{C}_{2 \mathrm{v}}\right)$ | $d_{67}$ | 1.575 | 77.125 (5.933) |
|  | $d_{51}$ | 1.588 |  |
|  | $d_{12}$ | 1.607 |  |
|  | $d_{56}$ | 1.626 |  |
|  | $d_{23}$ | 1.630 |  |
|  | $d_{16}$ | 1.662 |  |
|  | $d_{26}$ | 1.741 |  |
|  | $d_{27}$ | 1.765 |  |
| $\mathrm{c}\left(\mathrm{D}_{2 \mathrm{~h}}\right)$ | $d_{\text {min }}: d_{12}$ | 1.513 | 66.253 (5.096) |
|  | $d_{\text {max }}: d_{24}$ | 1.650 |  |
| $\mathrm{d}\left(\mathrm{C}_{2 \mathrm{v}}\right)$ | $d_{107}$ | 1.529 | 77.218 (5.940) |
|  | $d_{1113}$ | 1.532 |  |
|  | $d_{123}$ | 1.547 |  |
|  | $d_{41}$ | 1.575 |  |
|  | $d_{89}$ | 1.588 |  |
|  | $d_{45}$ | 1.645 |  |
|  | $d_{85}$ | 1.669 |  |
|  | $d_{56}$ | 1.686 |  |
|  | $d_{213}$ | 1.690 |  |
|  | $d_{52}$ | 1.777 |  |
|  | $d_{15}$ | 1.806 |  |
|  | $d_{59}$ | 1.811 |  |
| $\mathrm{e}\left(\mathrm{C}_{1}\right)$ | $d_{\text {min }}: d_{1213}$ | 1.541 | 76.292 (5.610) |
|  | $d_{\text {max }}: d_{23}$ | 1.773 |  |

$(1.60 \AA)$, that between the central atom and the four peripheral atoms is $1.77 \AA(1.83 \AA)$ and each of the other two apex atoms bonds to the three neighbouring atoms at a distance of $1.65 \AA$ $(1.70 \AA)$, where the values in parentheses are Boustani's results.

There is a chain structure of $B_{13}(c)$ similar to the structure of $B_{10}(o)$ in figure 2. Its binding energy per atom is $5.0964 \mathrm{eV}, 0.890 \mathrm{eV}$ less than that of $\mathrm{B}_{13}(\mathrm{~d})$.

## 4. Conclusions

For $\mathrm{B}_{7}$, the ground-state structure is $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}(\mathrm{a})$. There are three 3D structures. Their binding energies are $1.29 \mathrm{eV}, 1.43 \mathrm{eV}$ and 1.92 eV higher than that of $\mathrm{B}_{7}(\mathrm{a})$. The potential surface near $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{7}(\mathrm{a})$ and quasi- $\mathrm{D}_{6 \mathrm{~h}}-\mathrm{B}_{7}(\mathrm{e})$ is very flat.

For $\mathrm{B}_{10}$, the ground state is $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{10}(\mathrm{a})$, but the convex structure of $\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{10}$ is not stable in our MD calculation: it will automatically transform to $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{10}(\mathrm{a})$. There is a new quasi-planar structure, $\mathrm{B}_{10}(\mathrm{~b})$; it is only 0.70 eV higher than that of $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{10}$. There are twelve 3 D clusters in $\mathrm{B}_{10}$. Their total binding energies are 1.10 eV to 4.37 eV less than that of $\mathrm{C}_{2 \mathrm{~h}}-\mathrm{B}_{10}$.
$\mathrm{C}_{2 \mathrm{v}}-\mathrm{B}_{13}(\mathrm{~d})$ is the ground-state structure of the $\mathrm{B}_{13}$ cluster, but $\mathrm{B}_{13}(\mathrm{a})$ and $\mathrm{B}_{13}(\mathrm{~b})$ are almost degenerate with it.

Comparing the ground-state structures of $B_{7}, B_{10}$ and $B_{13}$, the $B_{13}$ cluster is almost planar, much flatter than $\mathrm{B}_{7}$. The bigger the cluster, the flatter it will be. In other words, the structure will get nearer to the ideal $\mathrm{sp}^{2}$ with increasing number of B atoms in the cluster.

It is suggested that the boron clusters grow according to the 'Aufbau principle'. Starting from the basic unit, the hexagonal pyramid $\mathrm{B}_{7}$, and adding atoms sequentially, one can form new hexagonal pyramids to obtain either quasi-planar or convex structures [7]. $\mathrm{B}_{10}$ has two hexagonal pyramids and $\mathrm{B}_{13}$ has three. In our final structures, $\mathrm{B}_{10}$ is a quasi-planar structure, but the convex structure is not stable. For $\mathrm{B}_{13}$, three degenerate ground states cannot be distinguished clearly as quasi-planar or convex structures, and $B_{13}(d)$ does not consist of three hexagonal pyramids. It seems that although the 'Aufbau principle' is very useful in describing the growth of boron clusters, the process is more complicated than that predicted by the principle.

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